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# Guidelines to select suitable parameters for contour method stress measurements

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## Abstract

The contour method is one of the promising techniques for the measurement of residual stresses in engineering components. In this method, the cut surfaces deform, owing to the relaxation of residual stresses. The deformations of the two cut surfaces are then measured and used to back calculate the 2-dimensional map of original residual stresses normal to the plane of the cut. Thus, it involves four main steps; specimen cutting, surface contour measurement, data analysis and finite element simulation. These steps should perform in a manner that they do not change the underlying features of surface deformation especially where the residual stress distribution varies over short distances. Therefore, to carefully implement these steps, it is important to select appropriate parameters such as surface deformation measurement spacing, data smoothing parameters ('knot spacing' for example cubic spline smoothing) and finite element mesh size. This research covers an investigation of these important parameters. A simple approach for choosing initial parameters is developed based on an idealised cosine displacement function (giving a self-equilibrated one-dimensional residual stress profile). In this research, guidelines are proposed to help the measurer to select the most suitable choice of these parameters based on the estimated wavelength of the residual stress field.

**Keyword** Contour method, deformation measurement spacing, knot spacing, mesh size, residual stresses.

## 1. Introduction

The contour method has emerged as a promising technique for the measurement of residual stresses in engineering components. This method was invented in 2000 by Mike Prime [1]. It is based on cutting the test component of interest in two halves. The cut surfaces deform, owing to the relaxation of residual stresses. The deformations of the two cut surfaces are then measured, and used to back calculate the 2-dimensional map of original residual stresses normal to the plane of the cut [2]. The contour method is capable of measuring through-thickness residual stresses. The contour method is relatively simple, inexpensive, and utilizes readily available equipment in workshops [3]. It has been successfully validated by commonly used residual stress measurement techniques, such as neutron diffraction [4], slitting [5], [6], synchrotron x-ray diffraction [7], [8] and sectioning [9]. The method is useful to obtain detailed information of residual stresses introduced by various manufacturing processes such as welding [3], [10]–[12], hammer peening [13], laser peening [14]–[16], cold expanded hole [17] and aluminium alloy forging [18]. Nevertheless, like the other residual stress measuring techniques, the contour method also suffers from factors that impact on the accuracy and the spatial resolution of the method, and cause uncertainties in the measured stresses. The reliability and accuracy of the contour method measurement results can be improved by minimising errors and uncertainties that can be introduced during data collection and data analysis procedures. The steps in undertaking the contour method of residual stress measurement are: specimen cutting, surface contour measurement, data analysis and Residual stress back calculation (FE modelling).

## **Specimen cutting**

Specimen cutting is the most crucial step of the contour method. Wire Electric Discharge Machining (WEDM) has previously been identified as the best choice for the cutting step of the contour method [2], [19], [20]. WEDM cutting is based on a thermo-electric process, and it is performed by generating a series of electrical sparks between the EDM wire (electrode), and the component [21]–[23]. It can be applied to all electrically conductive materials, irrespective of their hardness, material strength, shape and toughness. Also, WEDM is a non-contact machining process; there is no direct contact between the electrode and the work piece during cutting. Throughout the cutting process, the component is submerged in a temperature controlled deionized water tank, in order to minimise thermal effects from the cutting process.

## **Surface contour measurement**

After wire EDM cutting, the contours of the created cut surfaces are measured. Measuring the deviation from planarity of the cut surface, with appropriate surface deformation measurement spacing is important when using the contour method to get residual stress measurement results with high accuracy. A coordinate measuring machine (CMM) has the capability to register three spatial coordinates (displacements) for any point on a cut surface. CMMs can measure the cut surfaces using contact and non-contact devices, which include touch trigger probes, continuous scanning probes and optical system. The most common techniques for measuring the surface contours are reported in detail in [19]. A CMM, with a fitted touch probe [24]–[26], is the most commonly used instrument for taking surface contour measurements [1]. They are widely available in many engineering workshops. The measurement of surface displacement is used to quantify the residual stress values. Before conducting the CMM measurements, the cut surface must be clean and dry, and free of any dirt, dust and oil. Any dirt particles on the sample surface can affect the measurement data and can cause error in the contour method stress results. Since the touch probe sampling rate is about one measurement point per second the measurement process can take several hours. Therefore, temperature stability is important, so the contour cut surface is measured in a temperature controlled room and should be isolated from thermal fluctuation [2], [27]. Also, the touch probe makes contact with the measuring surface, and some local deformation occurs due to the low but finite contact force. These limitations can be overcome by using an entirely non-contact method such as laser sensors [28]. Due to faster acquisition of measurement points using laser sensors, they are more suitable for measurement of large engineering components. As such the thermal fluctuation, if there is any, is less of an issue. They have a capability to measure the cut surface with better resolution and high accuracy. However, laser sensors cannot exactly capture the outline of the cut surface perimeter because the outline of the cut surface needs to be measured in the transverse direction to the cut surface measurements and it is difficult to do with the laser scanners.

## **Data analysis**

The next step is to process the cut surface deformation data. To process the contour data, for the calculation of residual stresses using the contour method, several data analysis steps are involved. These steps include [19], [28]; aligning the contour data of the opposing cut surfaces, Interpolating the two data sets into a common grid, extrapolating to the perimeter, averaging the two sets of data points, smoothing the cleaned and averaged data, the following sections describe these steps in more detail.

### ***Aligning the data sets***

The two cut surface deformation data sets are measured in two different coordinate systems. These data sets must be aligned on the same coordinate system, so that all the points in both data sets are coincident with each other, in the same manner as the material points were in the single component prior to the cutting. The mating cut surfaces appear as mirror images of each other. In this situation, one of the  $x$ - $z$  coordinate directions needs translation and rotation,

so that both cut surfaces exactly overlay each other and the corresponding data points on each mating surface can be aligned. This data set alignment is facilitated by measuring the perimeter of both cut parts. Note that deformation measurement points are y coordinates and the points on the surface are on x-z plane.

### ***Interpolating in a common grid***

For several reasons, the data points of both cut surfaces cannot always be overlaid exactly on top of each other. **Reasons include; alignment of the cut surfaces and the defined local coordinates.** So, in this case, it is necessary to linearly interpolate the data sets of each cut surface onto a common grid, with the same approximate density, as the original measured data points [19], [28].

### ***Extrapolating to the perimeter***

The surface contour measurement method (CMMs and laser sensors) cannot exactly capture the displacement approaching the outline of the cut surface perimeter. Therefore, extrapolation is required to replace any missing data points, usually situated around the perimeter of the cut surface. This extrapolation is necessary because displacements must be applied to all the nodes on the cut surface in the FE model [19], [28]. Often reconstructed near surface residual stresses are unreliable and may not be reported.

### ***Averaging of the two data sets***

Once the measured surface data sets are aligned and on the same grid, they should be averaged point by point on the x-z grid to provide a single set of deformation data. This step is one of the most significant steps in the data processing because it can eliminate several potential sources of error, such as the effects of shear stresses and asymmetric cutting artefacts resulting from the cutting process [2], [19], [20].

### **Data smoothing**

**The averaged and cleaned displacement data set must be smoothed before using as boundary conditions in an FE model for elastic stress analysis. Data smoothing is required, because any variations within the contour cut surface data, resulting from the roughness on the WEDM cut surface, a cutting fault such as WEDM wire breakage, or an error in the surface contour measurement process, such as the CMM probe slipping at the edges of the cut surface, can be amplified in the stress results. If the overall form of the surface is to be preserved, it is essential to eliminate these irregularities. They can cause significant errors in the calculated stress values because for the contour method, stress calculation is dependent on surface displacement profiles [2], [19], [28].**

Surface data can be smoothed using different methods. Examples include bivariate spline smoothing, Fourier series and polynomial smoothing. The Fourier series method cannot always capture all the important features of the cut surfaces [28]. The most commonly used smoothing technique when using the contour method is bivariate spline fitting, or two dimensional (2D) cubic splines [28]. This technique, commonly used in previous contour measurement studies has led to the publication of very reliable results [29]–[31]. When using 2D cubic splines, piecewise polynomials are joined at given locations called ‘knots’ which define the domain of each polynomial. The smoothing process is achieved by minimising the uncertainty in the calculated stress results, or error in the data point and the fit. **The amount of smoothing and the density of knot spacing can affect the resulting stresses. For too small a knot spacing, the roughness of the cut surface can be incorporated into the final smooth surface contour data, and for too large a knot spacing, the final smooth surface contour data would not capture all underlying surface deformation features.** In both cases, the uncertainty in the calculated stresses would be increased. Hence, determining the optimum knot spacing, in order to obtain the best fit of the measured data, is essential to minimise uncertainty in the stress results.

Different approaches can be applied to determine the optimum smoothing parameter or ‘knot spacing’. Commonly, it can be achieved by fitting the measured displacement data to cubic splines with a variety of knot spacings. The suitability of the knot spacing is evaluated by comparing the spline fits to the raw data (averaged from both cut surfaces) [12], [31], [32]. Another approach to determine the optimum knot spacing, involves incrementally increasing the knot spacing, fitting the data for each knot spacing and then performing a finite element analysis for each increment to determine the stresses. The uncertainty in the calculated stresses at a given node is estimated by taking the standard deviation of the new stress and the stress from the previous, course fit. The standard deviation can be calculated from Eq 1.1.

$$\partial\sigma(i,j) = \frac{1}{\sqrt{2}} |\sigma(i,j) - \sigma(i,j-1)| \quad \text{Eq 1.1}$$

Where,  $\sigma(i,j)$  represents the stress at node  $i$  for the smoothing spline solution  $j$ , and  $j-1$  refers to the previous, course smoothing spline solution. An averaged uncertainty in the calculated stresses can then be calculated. The optimum knot spacing will always relate to the lowest average stress uncertainty using the root-mean-square (RMS) of all the nodal uncertainties from Eq 1.2 [28].

$$(\text{avg}) \partial\sigma(j) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^n [\partial\sigma(i,j)]^2} \quad \text{Eq 1.2}$$

### Residual stress back calculation (FE modelling)

For the contour method stress calculation, linear elastic finite element (FE) analysis is performed using a standard FE code such as ABAQUS. The contour cut has to be symmetric therefore only one of the cut halves is used to create a three dimensional finite element model. The model is created by using the measured perimeter of the cut part. Ideally the cut surface should be modelled with a deformed face and then forced back to a flat surface. However in practice, the measured deformations resulting from stress relaxation are very small in comparison to the size of the components being measured. Therefore, for convenience, the cut surface is modelled as having a flat (undeformed) cut face. The FE model of the specimen is meshed and the elastic material properties of the specimen are defined. The contour method is based on an elastic superposition principle. The material behaviour is assumed to have isotropic linearly elastic properties, defined by the values of Young modulus and Poisson’s ratio. Conventionally, the finite element model is meshed using brick elements, with either linear shape function hexahedral 8-node elements, or quadratic shape function hexahedral 20-node elements. The next step is to apply the smoothed data, in the form of displacement boundary conditions, on the FE nodes of the model, with reverse sign (i.e. the displacement contour is applied in the opposite direction). Then, additional boundary conditions are applied to FE model to prevent rigid body motion (see Figure 1) [33]. Finally, residual stresses are obtained by performing a linear elastic finite element analysis.

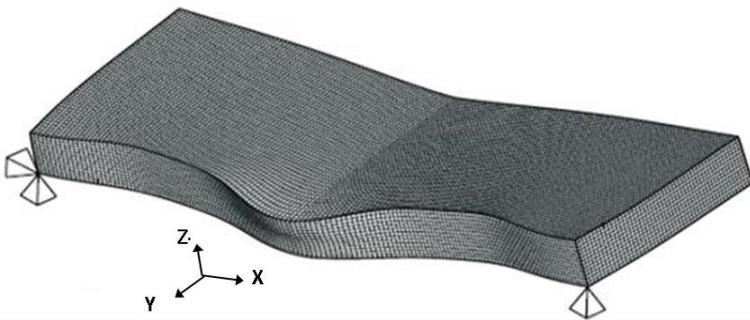


Figure 1: A deformed 3-dimensional linear FE model showing additional constraint to prevent rigid body motion [33].

In summary, for residual stress measurement using the contour method, deformation data defining the “contour” of the cut surface profile is applied to a finite element (FE) model of the cut component, and a linear elastic mechanical FE analysis is carried out to determine the residual stresses released by the cut. The following data collection and analysis parameters are important in this process:

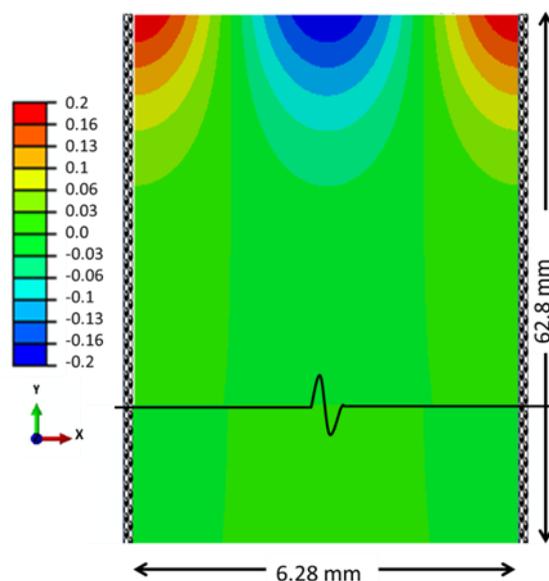
- The deformation measurement spacing of the cut face.
- The data smoothing (for example the ‘knot spacing’ in cubic spline smoothing).
- The size and type of element employed in the FE stress analysis.

A suitable choice of these parameters is essential, especially where the residual stress distribution varies over short distances. In this research the choice of parameters is studied by considering an idealised surface deformation profile and assessing how effectively the profile is captured using different sets of linear and cubic spline knot spacing intervals. The quality of fit is calculated from the error relative to the idealised profile. On the basis of this investigation, guidelines are provided to help contour method measurement practitioners select a suitable surface measurement density, knot spacing to smooth the deformation data and FE mesh size for the contour method data collection and analysis.

## 2. Idealised deformation profile

A cosine distribution of direct stress acting across a large plate is self-balancing and can therefore be taken to represent an idealised residual stress distribution. Consider a cosine displacement profile applied normal to the edge of a wide plate having a wavelength  $w$ , and peak amplitude  $M$ . The stress distribution at the surface is calculated for this case using a finite element (FE) stress analysis, for example with  $w = 6.28 \text{ mm}$  and  $M = 0.2 \text{ }\mu\text{m}$ , using symmetric boundary conditions and assuming plane strain conditions. The elastic material properties (Young’s modulus,  $E = 210 \text{ GPa}$  and Poisson’s ratio,  $\nu = 0.3$ ) are defined to obtain the residual stress distribution. *Figure 2* represents the FE model dimensions and boundary conditions. The FE stress results show that in the result of applied cosine displacement profile, the stress profile along the edge has a similar cosine form (see *Figure 3*). The following empirical formula (see in Eq 2.1) can be derived from the FE results.

$$\sigma\left(\frac{x}{w}\right) = 3.45 E \left(\frac{M}{w}\right) \cos\left(\frac{2\pi x}{w}\right) \quad \text{Eq 2.1}$$



*Figure 2: Finite element model, the boundary conditions and applied normal displacement ( $M = 0.2 \text{ microns}$ ) along the edge of a semi-infinite plate.*

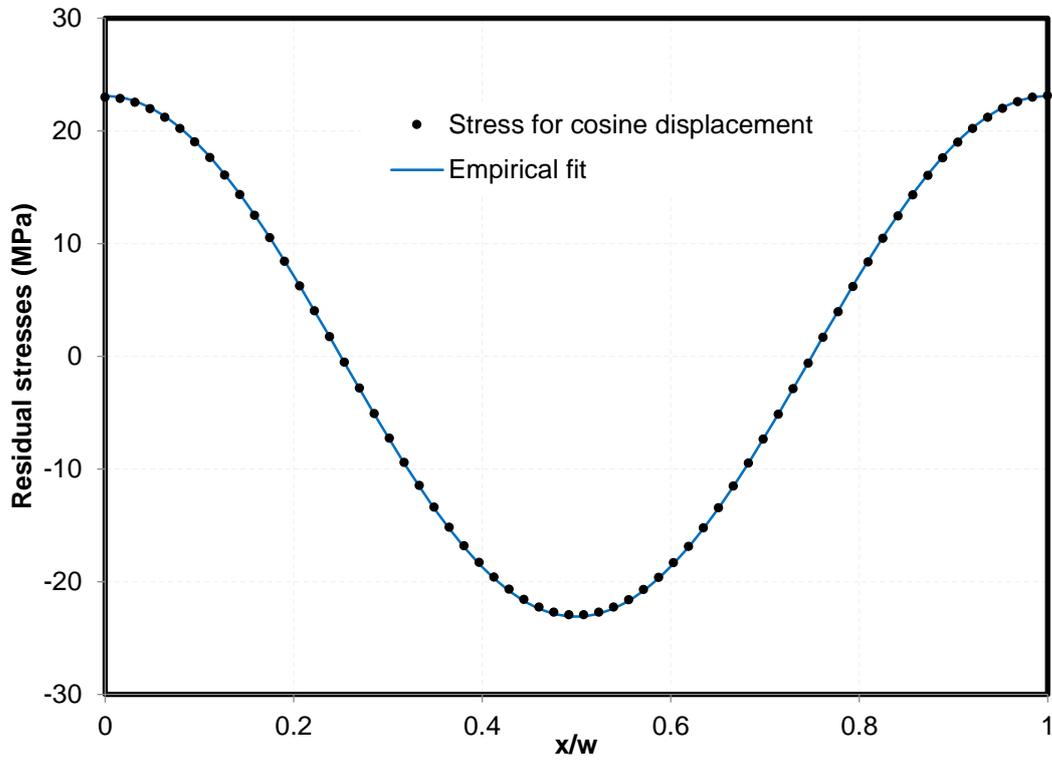


Figure 3: Predicted cosine form of self-equilibrated stress profile influence by a cosine displacement profile.

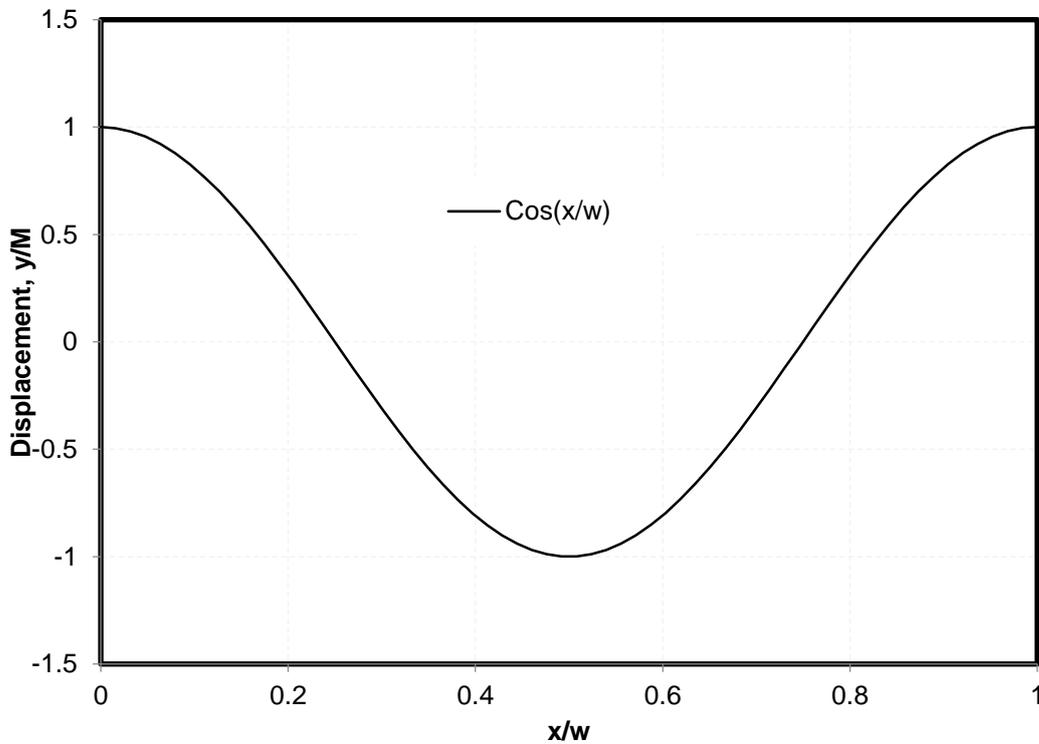


Figure 4: Idealised cosine displacement profile as a function of  $x/w$ .

The above study shows that an idealised one dimensional cosine surface deformation profile (see Figure 4) defined by Eq 2.2, can be used for simplified data analysis investigations.

$$y(x) = M \cos(n\phi) \quad \text{Eq 2.2}$$

Where,  $y(x)$  represents the surface deformation profile,  $M$  the maximum amplitude, and  $n$  is the order of the function and  $\phi = \frac{2\pi x}{w}$ , where  $w$  is the wavelength of the surface profile distribution.

For a simple case where  $M$  and  $n$  have values of 1, the cosine distribution  $y(x)$  has a period of  $2\pi$ , giving Eq 2.3.

$$y(x) = \cos\left(\frac{2\pi x}{w}\right) \quad \text{Eq 2.3}$$

### 3. Piece-wise linear fit to cosine deformation profile

The accuracy of piece-wise linear fits to a cosine displacement profile over different sets of spacings,  $a$ , ranging from  $a/w = 0.33$  to  $0.071$  (i.e.  $a = w/3$  to  $w/14$ ) are considered.

Figure 5 shows an example where five equally spaced sampling points are used, that is  $a = w/4$  for a cosine distribution. The plot also represents the piece-wise linear fit to sample points. Note that for this case sampling points start at  $x/w = 0$ .

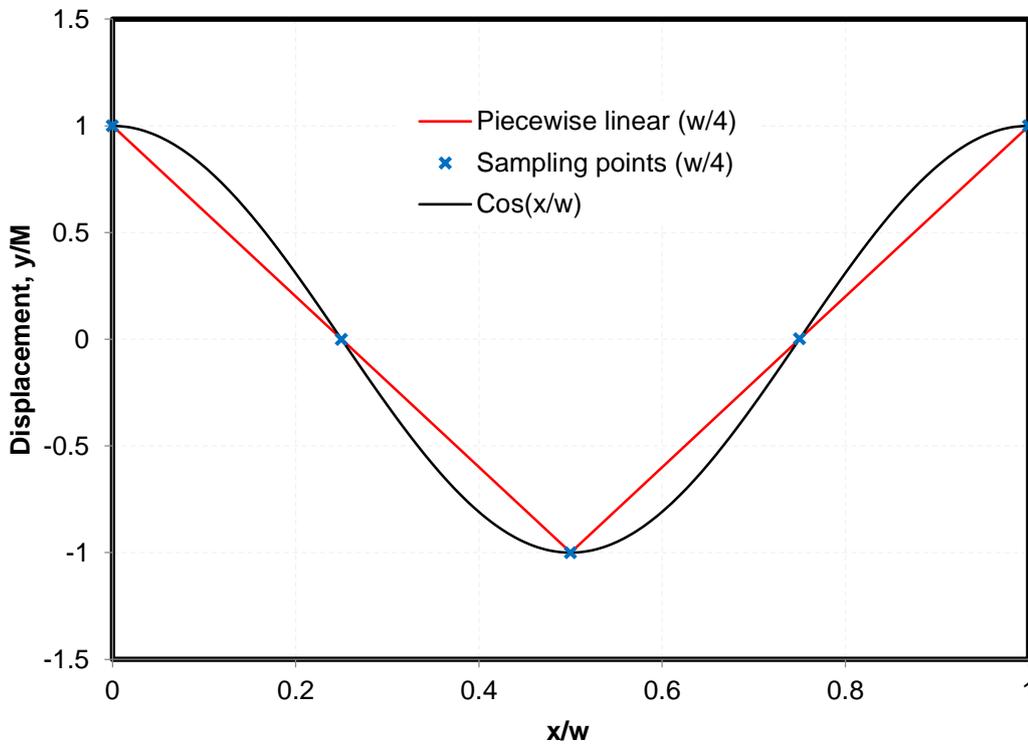


Figure 5: Piece-wise linear fit to sampling points spaced  $w/4$  apart on a cosine displacement profile.

The equation of a straight-line can give the piece-wise linear intermediate  $y$  values between two consecutive sampling points of each fit. Eq 3.1 represents the general equation for a straight-line.

$$(y - y_1) = m (x - x_1) \quad \text{Eq 3.1}$$

Where,

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$x$  and  $y$  represent the coordinate and ordinate respectively of the intermediate points between the two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , and  $m$  represents the gradient of the line. Eq 6 is used to

calculate the values of the  $y$  coordinate along the piecewise linear fit. The deviations (errors) of each piecewise linear fit to the idealised cosine displacement profile are then readily calculated. The modulus of deviations for each fit are determined and used to calculate the overall maximum deviation error, the mean deviation error and the root mean square (RMS) deviation error. The overall maximum deviation error is found directly by considering the largest value of the maximum deviations. The mean deviation error value is calculated by taking the mean of all the maximum deviation values of each fit. The RMS deviation error is calculated using Eq 3.2.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{N}} \quad \text{Eq 3.2}$$

Where:  $y_i$  is the  $y$  coordinate of the cosine profile at point  $i$  and  $\hat{y}_i$  is the  $y$  coordinate of the piecewise linear fit at point  $i$ .  $y_i - \hat{y}_i$  are the maximum values of the deviations of each piecewise linear fit.  $N$  is the number of intervals.

Then, the non-dimensional form of the maximum, mean and RMS error is calculated by normalising the maximum, mean and RMS errors to the idealised cosine displacement function. The normalized maximum, mean and root-mean square (NRMS) errors are defined in Eq 3.3.

$$N. \text{max or } N. \text{mean or } NRMS = \frac{\text{max or mean or } RMS}{y_{i(Max)} - y_{i(Min)}} \quad \text{Eq 3.3}$$

$y_{i(Max)}$  and  $y_{i(Min)}$  are defined by the maximum and minimum values of the cosine displacement function. The normalised maximum, mean and RMS values are represented as percentage errors (normalised mean and NRMS are multiplied by 100%). This procedure is repeated for the error calculations for all sets of spacing intervals.

Figure 6 demonstrates that the form of a cosine displacement distribution can be captured in a piece-wise linear manner with increasing error for  $a/w > 0.1$  (i-e  $a > w/10$ ). The maximum deviation error values vary from 2.3 % to 25 % and normalised mean and RMS deviation error values vary from 2 % to 18 %. For  $a/w = 0.1$ , the maximum deviation is  $< 2.5$  %, and the normalised mean and RMS deviations are  $< 2$  %; for  $a/w \leq 0.083$  ( $a \leq w / 12$ ) the maximum deviations are  $< 2$  %, and normalised mean and RMS deviations are  $\sim 1$  %. Taking  $\sim 1$  % as an acceptable NRMS error, it can be defined that a minimum of 12 equally spaced intervals must be selected ( $a/w \leq 0.083$ ).

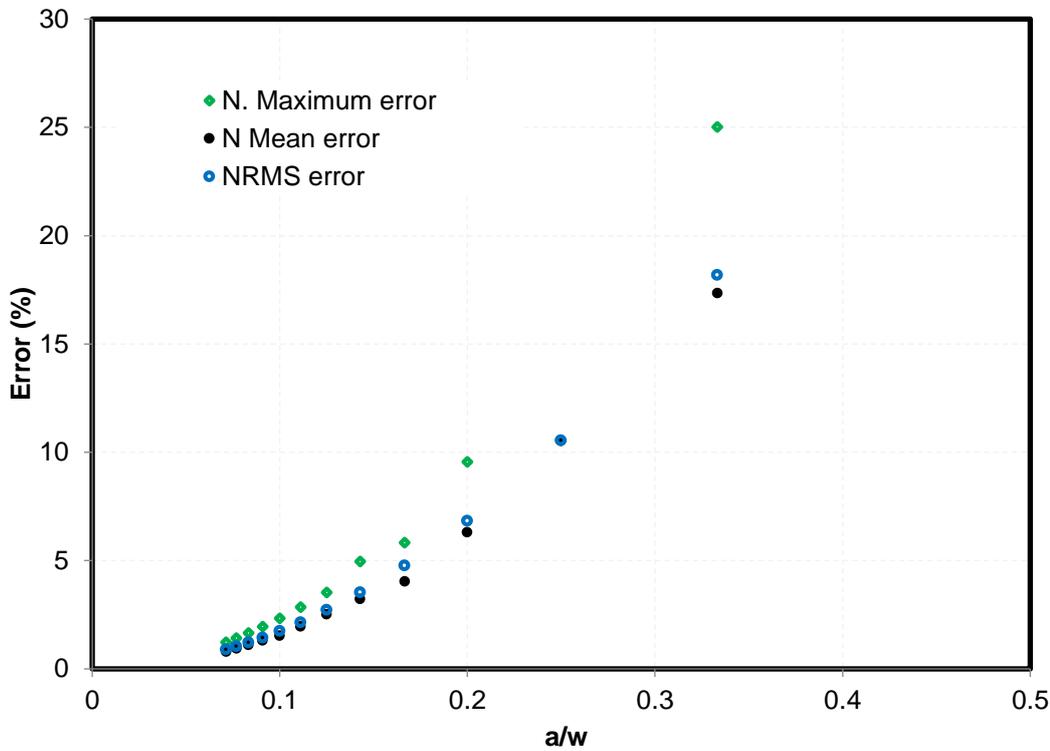


Figure 6: Error in piece-wise linear fits to cosine distribution as a function of spacing intervals  $a/w$ .

#### 4. Parameters for the contour method

##### Element mesh size (s) for contour stress analysis

A regular array of first order, linear hexahedral 8-node brick finite elements is commonly used to mesh the cut face of the finite element model in a contour measurement. First order brick elements of this kind represent constant stress in each element and have linear shape functions [34]. The errors introduced by idealising a simple cosine displacement function using first order elements (with linear variation in displacement from node to node) can be assessed using the errors analysis presented above (in section above Piece-wise linear fit to cosine deformation profile). Thus, at least 12 elements of constant size,  $s$ , are required to capture a cosine deformation profile of wavelength,  $w$ , that is  $s \leq w / 12$  ( $s/w \leq 0.083$ ) to ensure the NRMS error  $\leq 1\%$ .

##### Data smoothing (knot spacing, k)

In order to investigate the best choice for knot spacing to smooth the measured surface deformation data, errors associated with fitting an idealised function can be quantified in a similar way to the element mesh size study presented in section Element mesh size (s) for contour stress analysis. Cubic splines can be used to fit the idealised cosine displacement profile over different sets of knot spacing ranging from  $k/w$  0.33 to 0.071 (i.e.  $k = w/3$  to  $w/14$ ). In order to investigate the deviation between each spline fit and the original cosine displacement profile, the root-mean-squared (RMS), maximum and mean errors are calculated for each set of knot spacing ( $k$ ).

Figure 7 shows an example where 5 knots are used, that is  $k = w/4$  ( $k/w = 0.25$ ) to capture a cosine distribution. The plot also represents the spline fit between the knots.

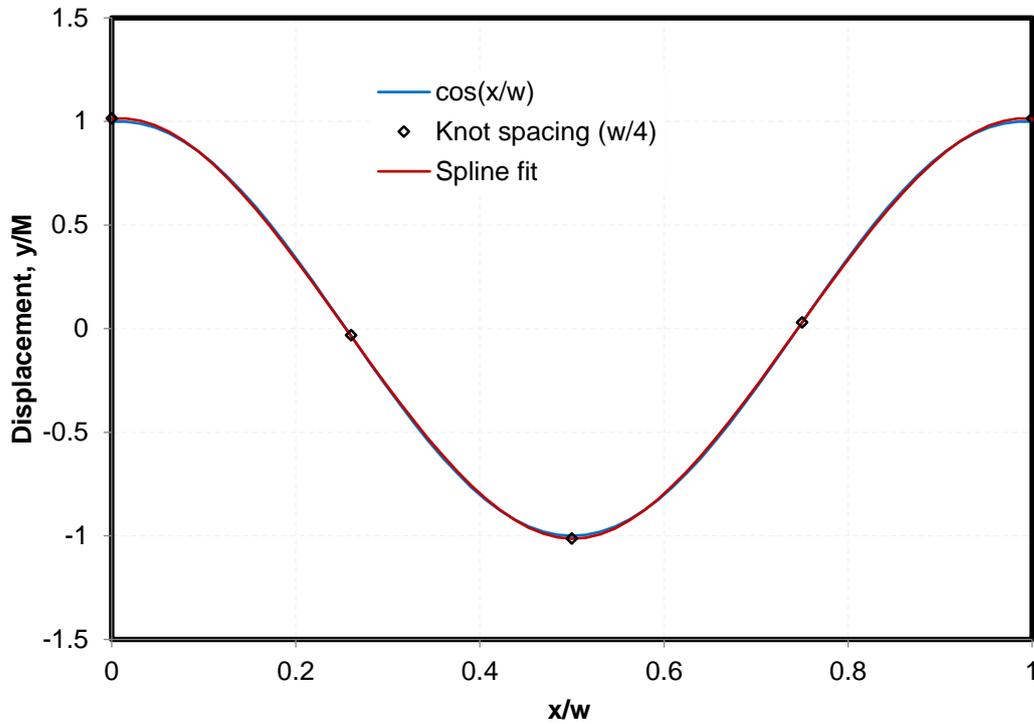


Figure 7: Spline fit to  $w/4$  knot spacing on a cosine displacement profile.

The root mean squared error (deviation) function is defined by Eq 3.2, but noting that here  $y_i$  are the y coordinates of the cosine profile representing the measurement data points,  $\hat{y}_i$  are the cubic spline fit data points and  $N$  is the total number of data points for each knot spacing interval. The maximum deviation error is found directly by considering the largest value of deviations. The mean deviation error value is calculated by taking the mean of all the deviation values of each fit.

The non-dimensional form of the RMS, maximum and mean errors are then calculated using Eq 3.2 given earlier, where  $y_{i(Max)}$  is the maximum displacement value taking from the idealised cosine distribution function, and  $y_{i(Min)}$  is the minimum displacement value taking from an idealised cosine distribution function. As previously the NRMS values are represented as percentage errors. This error is repeated for each set of knot spacings.

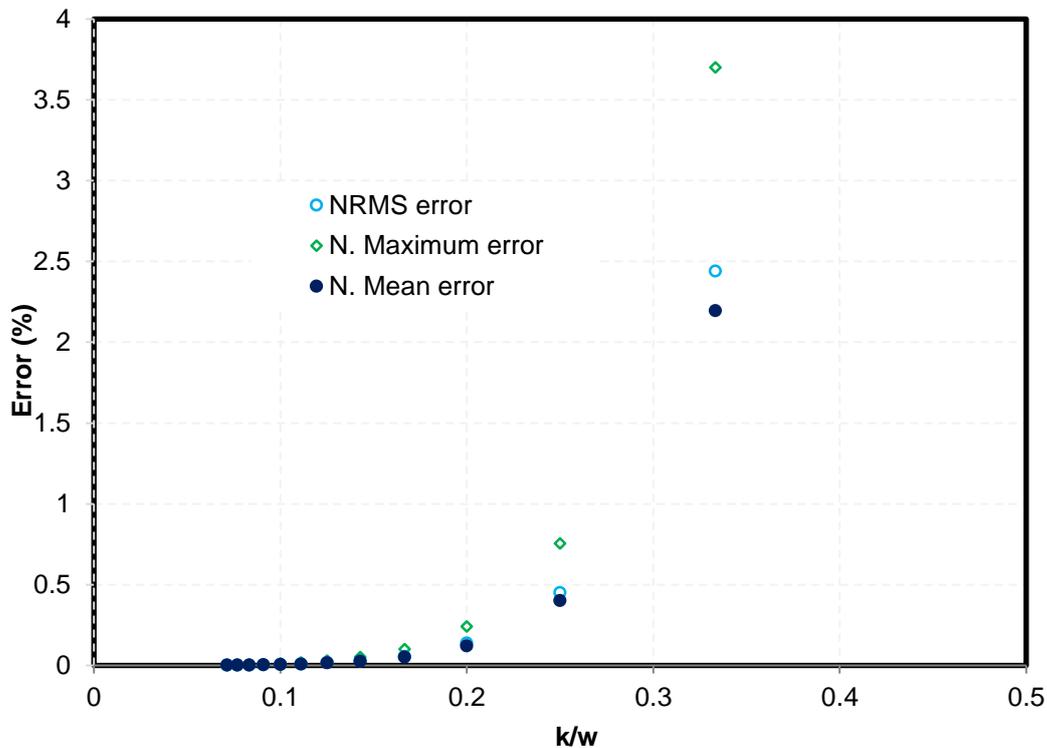


Figure 8: Error in cubic spline fits to a cosine distribution as a function of knot spacing interval ( $k/w$ ).

Figure 8 represents the error values versus knot spacing for the range of  $k$  intervals from  $k/w = 0.33$  to  $0.071$  ( $k = w/3$  to  $w/14$ ). Figure 8 shows that the percentage error increases with increase in the knot spacing. However, the errors are small because the spline fits efficiently capture the idealised cosine profile. From Figure 8 it is evident that knot spacings  $k/w \leq 0.25$  ( $k \leq w/4$ ) give a NRMS error, normalised mean error and normalised maximum error  $< 1\%$ . However, as the knot spacing increases the error begins to ramp up (for example for  $k/w = 0.33$ ). This evidence shows that 4 knot intervals can capture the idealised cosine displacement profile of wavelength,  $w$  with a NRMS error  $< 1\%$ .

### Surface deformation measurement spacing ( $d$ )

In order to acquire a good spline fit to the surface deformation profile introduced by the relaxed residual stress field a suitable surface measurement spacing  $d$  is required. The surface deformation of the cut face of a component in a contour measurement is usually measured in a regular grid of point spacing ( $d$ ) in both  $x$  and  $y$  directions as shown in Figure 9. For laser CMM measurements, each measured point is averaged over the laser beam diameter. For touch probe CMM measurements each measurement point represents the height of the surface area at which the probe makes the contact.

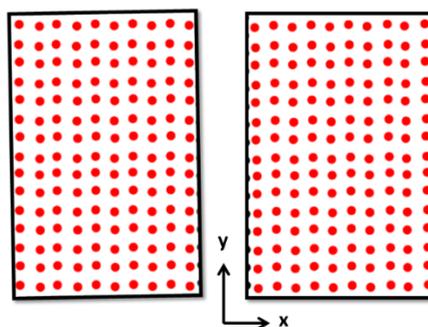


Figure 9: Schematic drawing showing a regular grid of surface deformation sampling points for the mating cut surfaces.

It can be intuitively argued that the measurement spacing,  $d$ , should be less than or equal to the linear element mesh size used to idealise the smoothed profile, that is  $d \leq s$ , where  $s \leq w/12$ . But ideally the measurement spacing should be as small as possible as several data points are required for cubic spline smoothing of noisy data between knots, that is  $d \ll k$ .

## 5. Residual stress wavelength, $w$

The residual stress wavelengths of interest have to be defined in order to apply the simple criteria developed above. A rigorous way of identifying the dominant cosine form wavelengths present in a residual stress field is to carry out a fourier series analysis [35]. But preliminary knowledge of the full residual stress field may not be available. Often the reason why a contour measurement is done is to actually quantify the residual stress field.

It is more difficult to measure short wavelength residual stress distributions because very fine surface deformations must be resolved as shown in Eq 2.1. The shortest residual stress length scale that can be resolved in a conventional contour method measurement can be inferred from the characteristic length scale of the surface roughness created by the WEDM process.  $RS_m$ , the mean spacing between the profile peaks, is an important surface roughness parameter as it provides a measure of the mean length scale of noise introduced by the cutting process. It has been estimated that the contour method is unlikely to be able to resolve variations in displacement across a length less than about five times  $RS_m$  [27]. The experimental results [36] show that,  $RS_m$  is  $\approx 0.15$  mm for a typical 0.25 mm diameter WEDM contour cut. Thus for this case the minimum residual stress length scale that can be practically measured by contour method is of order 0.5 to 1 mm.

More generally residual stress wavelengths likely to be present can be estimated using the following information:

- Prior knowledge: residual stress measurement results from other techniques, from prediction or/and published data from a similar component.
- Component dimensions (gives maximum wavelengths).
- Expert judgment.

## 6. Discussion

The criteria developed in this study can be used for choosing the measurement spacing  $d$ , cubic spline knot spacing  $k$  and finite element mesh size  $s$  for the contour method data analysis, providing the residual stress wavelengths of interest are known or can be estimated.

An appropriate estimation of the residual stress wavelengths of interest is essential because it has a great influence on establishing suitable choices for data analysis parameters. The developed criteria are based on a one dimensional idealised cosine displacement function of fixed wavelength and a simple estimation of errors. In practice, the contour method provides a two dimensional map of stress using surface deformation data measured across a two dimensional plane. Two dimensional cubic splines are used to smooth the deformation data and can provide better accuracy in the stress results. But the deformation field usually comprises a mixture of wavelengths including unwanted noise for which a more robust analysis is desirable. A further consideration is that deformation data are difficult to capture close to the edges of the specimen especially using a touch probe CMM [28]. But the edge effects have not been considered in the above study and again there is a scope for improving the criteria. The importance of selecting appropriate data analysis parameters becomes very high where short length scale residual stresses are of interest. But in order to resolve short length scale residual stresses, a very fine surface deformation measurement density is required for which an improved surface finish (lower roughness) is desirable to reduce 'noise' levels. In addition, the cut surface should be free from cutting effects. Therefore, a good quality of cut surface is essential for achieving a better resolution and accuracy in contour method residual stress results together with the suitable gauge size for data collection and data analysis parameters.

The gauge size for the contour method depends upon the spacing for the surface deformation measurements, the optimum knot spacing used to smooth the deformation data, and the element size used in the finite element stress analysis. The deformation of the cut surface should be measured using a suitable measurement spacing (which is usually smaller than the FE mesh size), and then the optimum knot spacing should be selected so that the associated cubic spline is best fitted to the displacement data. Then finally, first order elements are used to mesh the cut face of the finite element model for stress analysis. First order elements have linear shape function and provide constant stresses for each element. Thus, if first order elements are used the element size used at the cut surface gives a measure of the effective gauge size. Therefore, the effective 'top-hat' gauge size for the contour method can be controlled by FE element mesh size.

The following procedure is proposed to improve the reliability of contour residual stress measurements, especially where short length scale stress fields are of interest.

*Step 1:* Compile specimen geometry data and material mechanical properties including Young's modulus, Poisson's ratio and the material yield stress.

*Step 2:* Estimate the residual stress profile across the measurement plane from which the residual stress wavelengths  $w$ , of interest that best characterise the expected stress field can be identified. This can be obtained from other measurement techniques, from prediction or/and published data from a similar component.

*Step 3:* Perform WEDM contour cut using cutting conditions suggested in [19]. For short length scale residual stress variations, cutting conditions giving a fine surface finish should be chosen.

*Step 4:* Define the contour surface measurement density based upon the developed criteria; that is  $d \leq w/12$  and  $d \ll k$  noting that the finer the spacing the better.

*Step 5:* Measure the cut surface with defined sampling density.

*Step 6:* Perform data analysis steps.

*Step 7:* Choose the initial knot spacing for cubic spline smoothing based upon the wavelength analysis; that is  $k \leq w/4$ .

*Step 8:* Select the finite element mesh size based upon the wavelength analysis; that is  $s \leq w/12$ .

*Step 9:* Then, optimise the knot spacing using the uncertainty approach of Prime [28], by examining the different  $k$  spacings across the initial  $k$  value and calculate the stresses for each  $k$  increment. Estimate the averaged stress uncertainty for each  $k$  increment. The final  $k$  value is selected by minimising average uncertainty in the calculated stresses.

*Step 10:* Perform final FE analysis to calculate stress results.

All above steps are summarised in the flowchart shown in *Figure 10*.

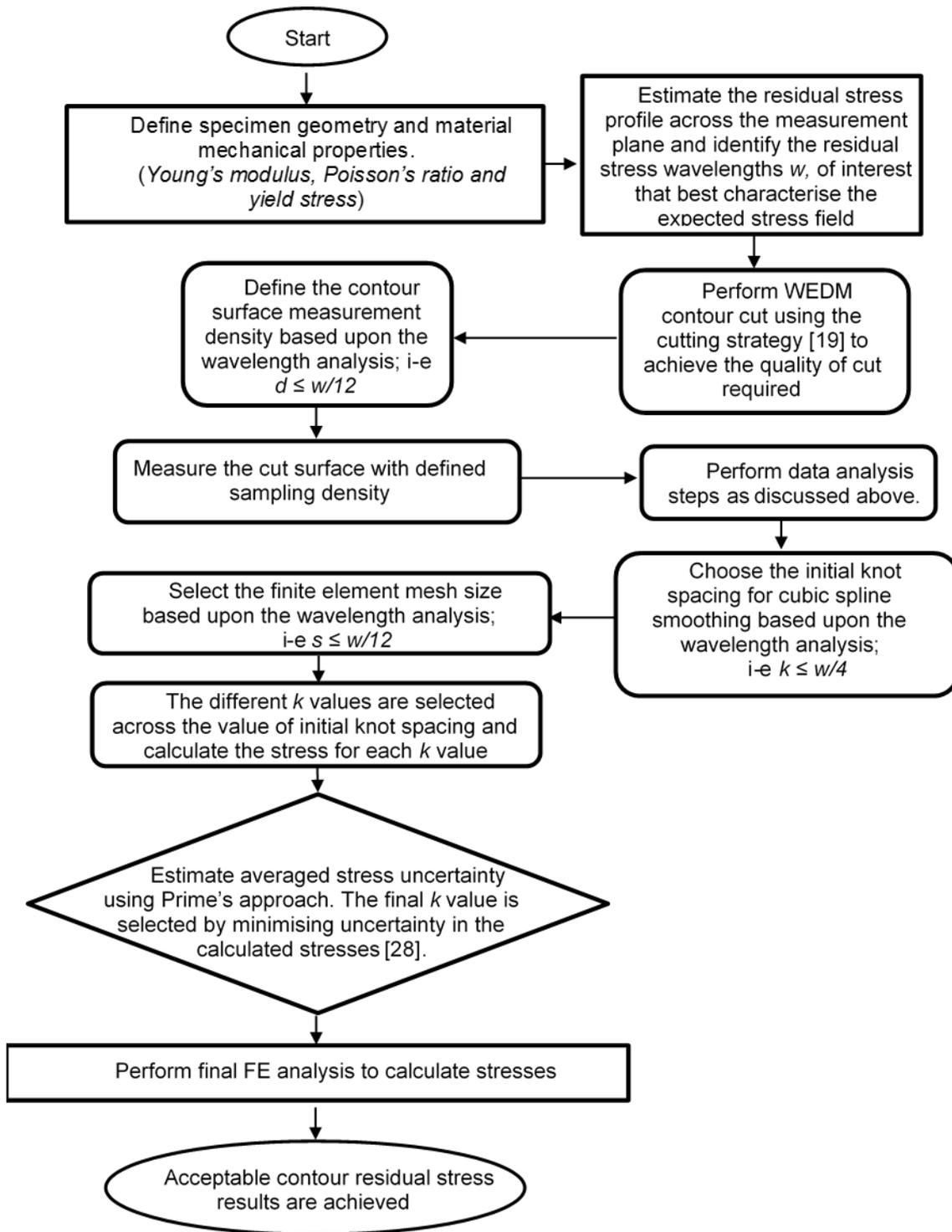


Figure 10: Flowchart illustrating the proposed contour method data analysis procedure to improve the robustness of the calculated results.

## Conclusion

- Three deformation data collection and analysis parameters have a major influence on the contour method residual stress results: the surface deformation measurements spacing,  $d$ , the cubic spline knot spacing,  $k$ , chosen to smooth the measured deformation and the finite element mesh size,  $s$ .
- The contour method data collecting and analysis parameters have been investigated by considering a one dimensional idealised cosine function. The quality of piece-wise linear and cubic spline fits to the idealised profile have been evaluated by calculating the fitting errors. Threshold acceptable errors are defined which inform the choice of these parameters.
  - The residual stress wavelength,  $w$ , likely to be present in the specimen is first needed to apply the simple developed criteria.
  - For the measurement spacing, select  $d \leq w / 12$  ( $d/w \leq 0.083$ ) and  $d \ll k$  noting, the finer the spacing the better.
  - For the knot spacing, select  $k \leq w/4$  ( $k/w \leq 0.25$ ).
  - For the finite element mesh size, select  $s \leq w / 12$  ( $s/w \leq 0.083$ ).

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